Data-adaptive approximation strategy for monitoring and analysis of height changes

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Motivation

- extensive dewatering for the mining of lignite
- consequence is extensive subsidence of ground
- monitoring of towns with high precision leveling
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Motivation

- extensive dewatering for the mining of lignite
- consequence is extensive subsidence of ground
- monitoring of towns with high precision leveling
- position of measured height changes due to infrastructure
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- monitoring of towns with high precision leveling
- position of measured height changes due to infrastructure

**target**: parametrization of height changes

1. how to parameterize and approximate the height changes?
2. how to judge the quality of the underlying observation network (distribution and number of surveying points)?
Data-adaptive approximation of height changes

Optimization of existing observation networks

Verification by independent control observations

Conclusion
Data-adaptive approximation of height changes

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Conclusion
Approximation of height changes

Basics

▸ model of bivariate polynomial

\[ l_i = P(x_i, y_i) = \sum_{k=0}^{m_x} \sum_{a=0}^{m_y-k} p_{k,a} \cdot x_i^k \cdot y_i^a \]

▸ adapt orders \( m_x \) and \( m_y \) to complexity of height changes

▸ elimination of non-significant parameters \( p_{k,a} \)

▸ detection of outliers

▸ checking of adjustments quality by global test

Parameter estimation

▸ functional model: \( l + v = Ap \)

▸ stochastic model: \( \Sigma_{ll} = \sigma^2 I \), with \( \sigma = 1 \text{mm} \)

\[ \hat{p} = \left( A^T \Sigma_{ll}^{-1} A \right)^{-1} A^T \Sigma_{ll}^{-1} l \]
Approximation of height changes

- orders: \( m_x = 6, m_y = 6 \)
- number of parameters: 23
- number of eliminated parameters: 5
- number of points: 194
- number of outliers: 15
- global test: accepted

- approximation successful
Questions

▸ number of surveying points sufficient?
▸ surveying points distributed homogeneously enough?

Consequences

▸ where to add points?
▸ where to save points?

⇒ analysis and optimization of observation network
Data-adaptive approximation of height changes

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Optimization of observation networks

1st approach: sampling theorem

building of regular grid

2nd approach: partial redundancies

validation of regular grid
Sampling theorem used in time series analysis

- Nyquist frequency: minimal 2 observations per wave to detect its frequency
- assumption: no noise, endless signal
- practical investigations: 5 observations per wave
1st approach: sampling theorem

Transfer to optimization of networks

(1) order of polynomial: \( m \)

(2) # waves: \( n_w = \frac{m - 1}{2} \)

(3) # points: \( n_g = \lceil 5 \cdot n_w \rceil \)

(4) both, in \( x \) and \( y \)-direction

(5) # homogeneously distributed points: \( n_{opt} = n_{g,x} \cdot n_{g,y} \)

\[ \Rightarrow \text{order of polynomial} \Rightarrow \# \text{ waves} \Rightarrow \# \text{ points} \]
2nd approach: partial redundancies

Parameter estimation

- $l_i = \sum_k \sum_a p_k a x_i^k y_i^a$
- $l + v = Ap, \quad \Sigma_l = \sigma^2 I$
- $\hat{p} = \left( A^T \Sigma_l^{-1} A \right)^{-1} A^T \Sigma_l^{-1} l$
- $\hat{l} = A \left( A^T \Sigma_l^{-1} A \right)^{-1} A^T \Sigma_l^{-1} l$

Partial redundancies $r_i$

- impact factors: $h_i = H_{i,i}$
- partial redundancies: $r_i = 1 - h_i$
- controllability of an observation
1. data-adaptive grid
   - build grid proportional to complexity of height changes
   - 5 points per wave
   - optimize grid until 1 point per cell
2. data reduction
   - eliminate points with $r_i > 0.95$
3. reinsert boundary points
   - due to limited dimensions

<table>
<thead>
<tr>
<th></th>
<th>$x$</th>
<th>$y$</th>
</tr>
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<tbody>
<tr>
<td>$m$</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$n_w$</td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>$n_g$</td>
<td>5</td>
<td>7.5</td>
</tr>
</tbody>
</table>
Example A

- orders $x:6, y:6$
- # points: 179
  - to save: 95
  - remaining: 84
  - to add: 12
- optimal number: 96
Example B

- orders $x:6$, $y:4$
- # points: 163
  - to save: 83
  - remaining: 80
  - to add: 7
- optimal number: 87
Findings

- # points: 163
  - to save: 83
  - remaining: 80
  - to add: 7
- optimal number: 87

Questions

- How to verify the optimization?
- How to verify the general approximation?
- What to do with the saved observations?

⇒ verification by saved observations (independent control observations)
Data-adaptive approximation of height changes

Optimization of existing observation networks

Verification by independent control observations

Conclusion
until now...

- only **remaining** points are used for approximation of the height changes
- **saved** points are not used at all

from now on...

- using **saved** points as independent control observations
- insertion of further points as substitution for the **missing** points
step 1: verification by residuals

- using **remaining** points for approximating polynomial model
- analyzing residuals of **saved** points not used for approximation

residuals of saved points inconspicuous (< 2.58 mm)
step 2: verification by model

- using all points (remaining + saved) for approximating polynomial model 1
- using remaining points for approximating polynomial model 2
- analyzing difference between models 1 and 2

- no significant difference (> 2.58 mm) between models
Independent control observations

- residuals of saved points inconspicuous
- no significant difference between models

- following ...
- 1. optimization procedure verified
- 2. approximation of height changes verified
Data-adaptive approximation of height changes

Optimization of existing observation networks

Verification by independent control observations

Conclusion
Data-adaptive approximation of height changes
  ▶ reliable and accurate analysis of subsidence of ground
  ▶ reliability and accuracy ensured by strict evaluation of approximation
    (not shown here, see references)

Optimization of existing observation networks
  ▶ estimating optimal number of observations
  ▶ analyzing where to add and where to save observations

Verification by independent control observations
  ▶ verification of optimization procedure
  ▶ verification of data-adaptive approximation of height changes
! Thanks for your attention!

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